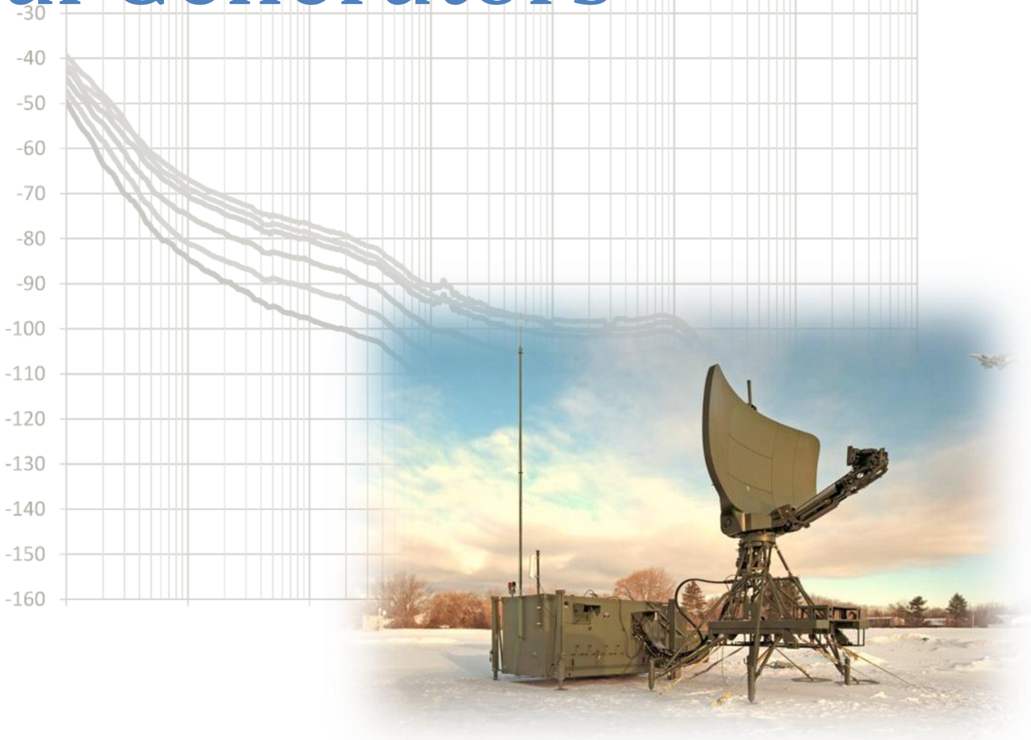
White Paper

Introduction to Phase Noise in Signal Generators



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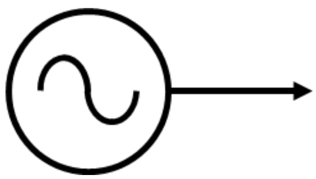
Phase Noise is an Attribute of All Real World Signals

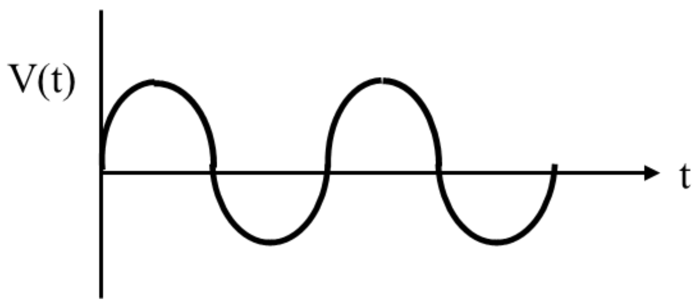
Phase noise is the result of small random fluctuations or uncertainty in the phase of an electronic signal. We specify and measure phase noise because it is a fundamental limitation in the performance of systems, limiting dynamic range. This shows up in radar and communications as loss of sensitivity, in imaging as lack of definition and in digital systems as higher bit error rate. While this discussion will focus primarily on phase noise in the frequency domain, phase noise can also be quantified as jitter in the time domain.

Most electronic signals derive from oscillators.



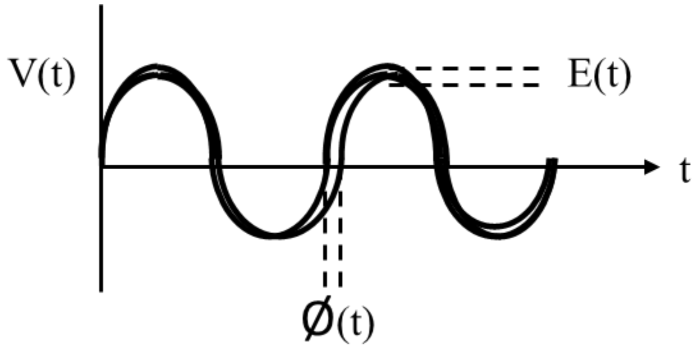
V(t) is the oscillator output in volts versus time at the oscillator frequency.

We can describe an ideal signal as $V(t) = A_o \sin 2\pi f_o t$ where A_o is the nominal amplitude and f_o is the nominal frequency.

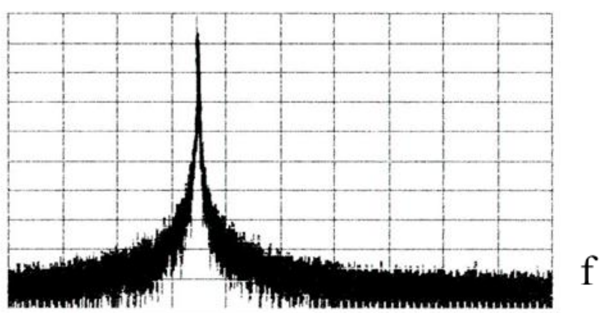


In the frequency domain, an ideal sine wave signal is shown as a single frequency in the spectrum.

We can describe a "Real-world" signal as $V(t) = [A_o + E(t)] \sin [2\pi f_o t + \text{Ø}(t)]$ where E(t) is the random amplitude fluctuations and Ø(t) is the random phase fluctuations.



In the frequency domain, this signal now appears as the familiar spectrum of carrier with sidebands.



It is important to point out that the amplitude noise and phase noise are small perturbations. The goal in signal generator design is to have these as small as possible, from at least 40 dB less than the carrier to as much as 170 dB less than the carrier, limited only by the kTB thermal noise floor.

Phase Noise in Signal Generators

Phase noise is quantified in the unit of measure $\mathcal{L}(f)$ called "script L of F", where

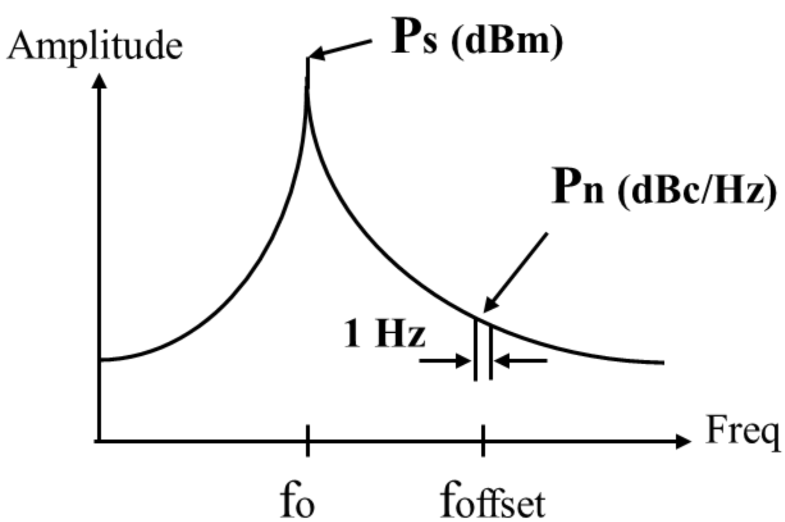
$\mathcal{L}(f)$ = *single sideband power referenced to the carrier*

- In a 1 Hz bandwidth at a frequency f Hz away from the carrier
- Divided by the carrier signals' total power

$$\mathcal{L}(f) = \frac{\text{Noise Power in 1Hz bandwidth}}{\text{Carrier Signal Power}}$$

$\mathcal{L}(f)$ = Pn (dBc/Hz) – Ps (dBm) where Ps is the carrier power.

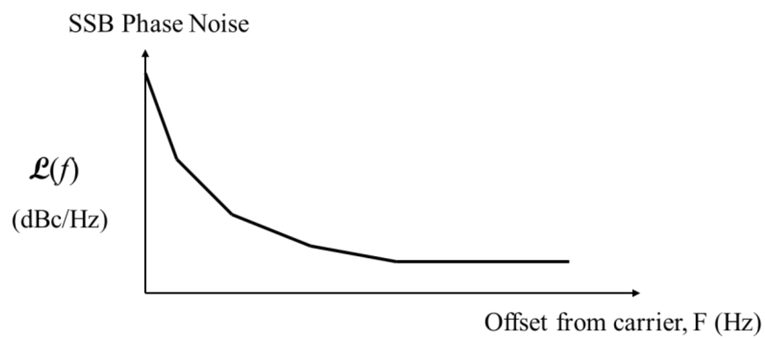
$\mathcal{L}(f)$ has units of dBc/Hz.



Measured directly on a microwave spectrum analyzer, $\mathcal{L}(f)$ is the ratio of the noise power in a 1 Hz bandwidth, at a specified offset from the carrier, to the carrier signal power.

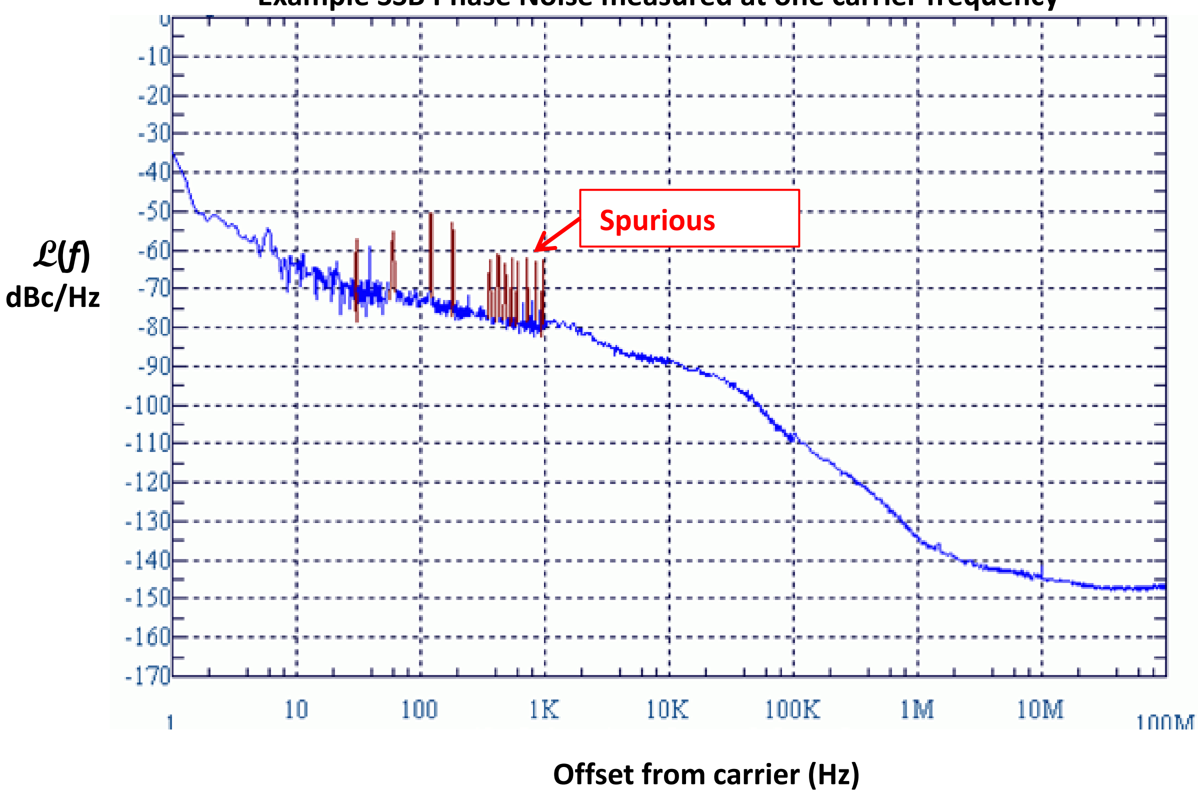
In the literature, the term $S_\phi(f)$ is often used to describe the two-sided "spectral density" phase noise. There are other terms, but these two are the most common. In product performance specifications, the $\mathcal{L}(f)$ single sideband phase noise is the industry standard. The two are related by the equation:

$$\mathcal{L}(f) = \frac{S\phi\,(f)}{2}$$



$\mathcal{L}(f)$, the single sideband (SSB) phase noise, is graphed as amplitude versus the frequency offset.

Example SSB Phase Noise measured at one carrier frequency



The above example is a typical log-log plot of signal generator phase noise, as measured on a phase noise test set. The vertical axis is amplitude relative to the carrier, which is not shown. The horizontal axis is frequency offset from the carrier. The scale will vary depending on both phase noise test set limitations and on what the signal generator manufacturer wants to highlight.

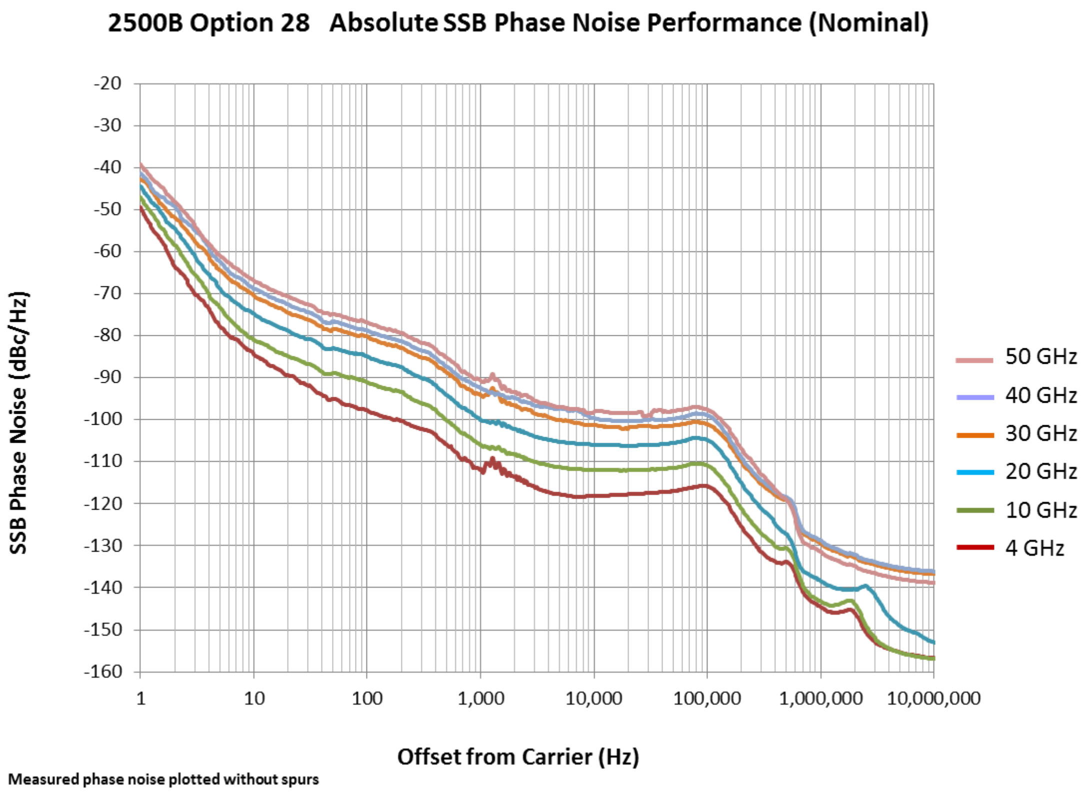
This example is for one frequency of the carrier, but it is common to see multiple traces for different carrier frequencies shown on one graph. You read the value of phase noise from the curve as the value at the offset (at that carrier frequency). From this example, the phase noise is -110 dBc/Hz at 100 kHz offset (at that carrier frequency).

In the literature, it is common practice to refer to the values of phase noise as “close-in” for the frequency offset range of 1 Hz to 100 Hz. That is, “close in phase noise” refers to the phase noise close to the carrier, less than 1 kHz away. Similarly, “far out” phase noise commonly refers to values 1 MHz or more from the carrier. That is, “far out phase noise” refers to the phase noise far from the carrier, more than 100 kHz away. The mid-range region, 1 kHz to 100 kHz offset, especially for signal generator phase noise, is sometimes referred to as the pedestal region.

The example above also is shown with spurious signals (in red). These spurious signals are not phase noise. They are not random, but systematic of any given signal generator. While many phase noise test sets will measure both phase noise and spurious signals together, most often the phase noise measurements are shown without the spurious signals. While spurious signals are always present in every signal generator, they have separate specifications, behave differently and removing them from the phase noise plot is a means of separating deterministic from non-deterministic artifacts.

A side note: while consistently shown as the upper sideband, it is generally agreed that the noise of the upper and lower sidebands are equal.[1]

[1] “Correlation Between Upper and Lower Noise Sidebands” by F.L. Walls, NIST, 1998



The above example is measured phase noise performance of the Giga-tronics 2500B Microwave Signal Generator, with curves for six different carrier frequencies, plotted without the spurious signals. Note that the phase noise increases as the carrier frequency increases, while holding the same general shape of the curve.

Phase Noise Limits in Signal Generators

Thermal noise, something referred to as “white noise” is broadband and flat with frequency, and is expressed by the formula:

N_T = kTB where k = Boltzman’s constant, T = temperature in Kelvin and B = bandwidth

For T = 290K (~ 17 °C), N_T = -174 dBm/Hz

In the literature, it is generally agreed that this is equally phase noise and AM noise, with the phase noise level and AM noise level = -177 dBm/Hz

Note that there may be other factors besides kTB that will raise this lower limit. As an example of how easily this number increases at microwave frequencies, look at the noise level out of a microwave amplifier with 10 GHz bandwidth (100 dB), 40 dB of gain and 10 dB of noise figure:

N_T = -174 + 100 + 40 + 10 = -24 dBm

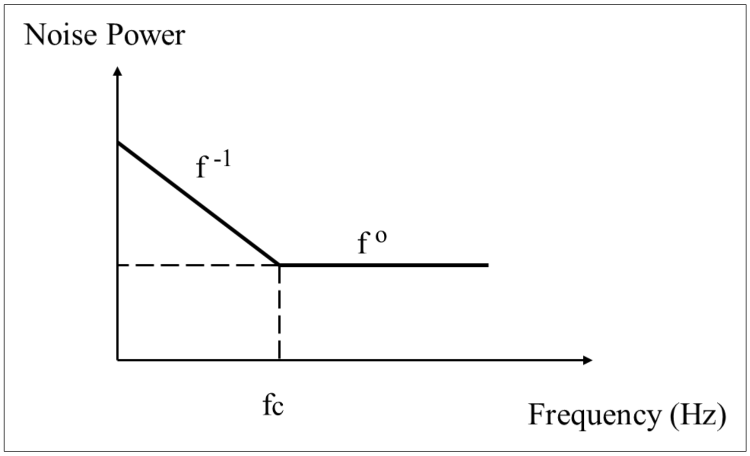
Now, going back to our phase noise definition, $\mathcal{L}(f)$ = Pn (dBc/Hz) – Ps (dBm) , the thermal noise limits phase noise as the power of the carrier drops:

Ps (dBm)	$\mathcal{L}(f)$ (dBc/Hz) limit
0	-177
-10	-167
-20	-157

This is why oscillator designers try to get as much power out of the oscillator as possible, since subsequent amplification will not improve the phase noise.

While thermal noise is flat (f^0) with frequency, “real-world” noise is not. Random noise in electronic systems increases near the carrier. “Flicker” noise or “pink” noise has a slope of f^{-1} versus frequency. This “1/f” or “one over f” noise is attributed to electron motion and is a naturally occurring phenomena not limited to electrical noise. (There are many papers in the literature focused on 1/f noise).[2,3]

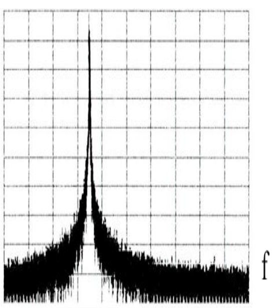
[2] See “Bibliograph on 1/f Noise” by Wentian Li, www.nslij-genetics.org/wli/1fnoise
[3] See “1/f noise: a pedagogical review” by Edoardo Milotti

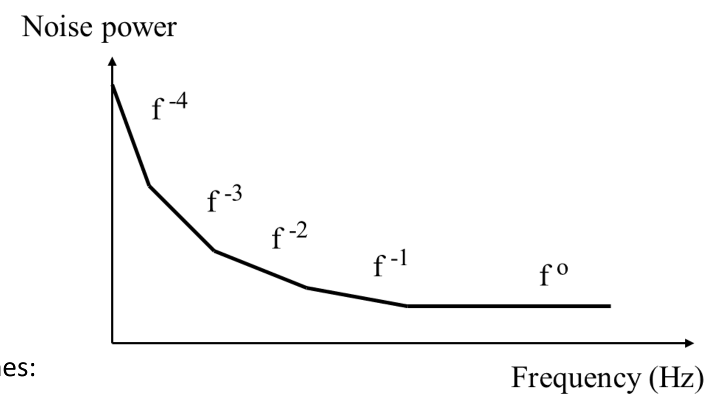


F_c is called the corner frequency and varies depending upon the technology.

This noise power characteristic applies to transistors and other active devices. When a transistor or other active device is incorporated into a feedback oscillator circuit, the f^0 and f^{-1} phase fluctuations convert to frequency fluctuations, and the buffer amplifier adds its own f^0 and f^{-1} noise to the signal. The frequency fluctuations exhibit f^{-2} and f^{-3} behavior.[4]

These noise factors combine to form the familiar signal curve shape seen on a spectrum analyzer.





These various slopes have been given names:

f^0 = White phase noise

f^{-1} = Flicker phase noise (10 db/decade)

f^{-2} = White FM (Random walk PM) (20 dB/decade)

f^{-3} = Flicker FM (30 dB/decade)

f^{-4} = Random walk FM (40 dB/decade)

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[4] “Characterization of Clocks and Oscillators” by D.B. Sullivan, D.W. Allan, D.A. Howe and F.L. Walls, NIST Technical Note 1337, 1990

The design of a low phase noise microwave signal generator requires careful attention to all the factors that may contribute to the noise, from the active circuitry and choice of components to filtering power supplies, isolating and shielding circuits, and shock mounting fans.

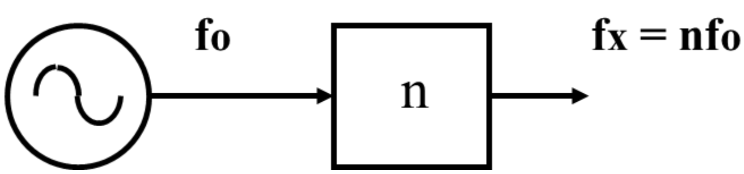
Regarding AM noise, from our definition, $V(t) = [A_o + E(t)] \sin [2\pi f_o t + \text{Ø}(t)]$ where E(t) is the random amplitude fluctuations, we can define $N_{AM}(f)$ = single-sideband AM noise in dBc/Hz:

$$N_{AM}(f) = \frac{\text{Spectral density of one AM modulation sideband}}{\text{Total sideband power}}$$

AM noise is the power spectral density of amplitude noise in a one Hertz bandwidth relative to the carrier power. AM noise does not peak around the carrier as phase noise does, so is negligible close to carrier. Far from carrier, at offsets typically greater than 1 MHz, AM noise starts to dominate. The broadband noise floor far from carrier (> 10 MHz offset) is primarily AM noise power, sometimes express as dBm/Hz rather than in dBc/Hz.

Effect of Frequency Multiplication on Phase Noise

Most microwave signal generators use frequency multiplication (and division) to achieve their wide frequency range. What is the phase noise of frequency multiplied signals?



F_0 is the carrier frequency and n is the multiplier or divider value, normally an integer.

F_x is the desired multiplied (or divided) frequency, equal to n times f_0.

Starting with our definition, $V_0(t) = \sin [2\pi f_o t + \text{Ø}(t)]$, then $V_x(t) = \sin n[2\pi f_o t + \text{Ø}(t)]$.

Δf is the frequency fluctuation of $V_0(t)$ and n Δf is the frequency fluctuation of $V_x(t)$.

$$\mathcal{L}(fo) = 20 \log \left(\frac{\Delta f}{f_{offset}} \right) \qquad \frac{\mathcal{L}(fx)}{\mathcal{L}(fo)} = 20 \log n$$

$$\mathcal{L}(fx) = 20 \log \left(\frac{n\Delta f}{f_{offset}} \right) \qquad \mathcal{L}(fx) = \mathcal{L}(fo) + 20 \log n$$

The result, $\mathcal{L}(f_x) = \mathcal{L}(f_0)$ + 20 log n, is not completely general, but is widely accepted for signal generator phase noise, where frequency multiplication and division is common practice.

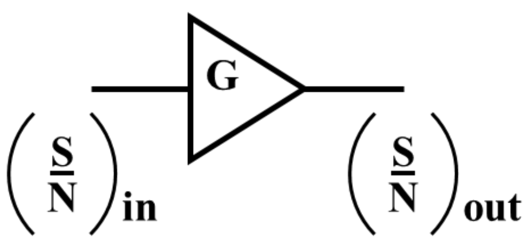
The phase noise increases when a signal is frequency multiplied, +6 dB for every doubling.

The phase noise decreases when a signal is frequency divided, -6 dB for every division by 2.

A note of caution, the effect on AM noise is a function of the multiplier circuit, and more importantly, the multiplier circuit may contribute AM to PM noise. In addition, there may be some small amount of residual noise added as well. The result is that the true phase noise may be slightly higher than predicted by the equation

Relating Amplifier Noise Figure to Phase Noise

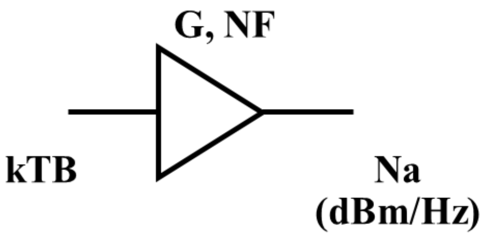
Most microwave signal generators use power amplifiers to achieve their output power performance. Or external microwave power amplifiers are added to boost output power. What is the phase noise of amplified signals?



An amplifier with gain G and noise figure defined in terms of signal-to-noise ratio:

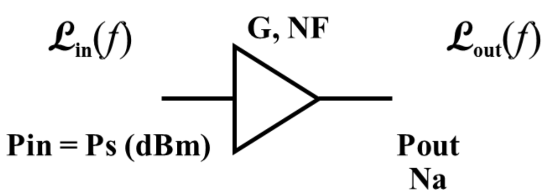
$$NF_{dB} = 10 \log \left. \frac{(S/N)\ in}{(S/N)\ out} \right|_{Ts = 290\ °K}$$

The noise power at the output of the amplifier can be calculated if the gain and noise figure are known.



$$Na = \text{NF (dB)} + \text{G (dB)} + \text{kTB (dBm/Hz)}$$

apply a carrier signal with Pin = Ps (dBm). $\mathcal{L}_{in}(f)$ is the phase noise of the applied signal Pin. $\mathcal{L}_{out}(f)$ will be the phase noise of the amplified signal.



$$\text{Pout} = \text{Ps (dBm)} + \text{G (dB)}$$

$$\mathcal{L}_{out}(f) = \mathcal{L}_{in}(f) + \text{Na (dBm/Hz)} - \text{Pout (dBm)}$$

$$\mathcal{L}_{out}(f) = \mathcal{L}_{in}(f) + \{\text{NF (dB)} + \text{G (dB)} + \text{kTB (dBm/Hz)}\} - \{\text{Ps (dBm)} + \text{G (dB)}\}$$

$$\mathcal{L}_{out}(f) = \mathcal{L}_{in}(f) + \text{NF (dB)} + \text{kTB (dBm/Hz)} - \text{Ps (dBm)}$$

The phase noise is increased by the noise figure of the amplifier, but the effect of the gain cancels out. That is to say, an ideal amplifier would amplify both the signal and phase noise equally, while a “real-world” amplifier will add noise to the signal. The phase noise is directly proportional to the thermal noise at the input and the noise figure of the amplifier.[5,6]

[5] “Design and Characterization of Low Phase Noise Microwave Circuits” by Jason Breitbarth, PhD thesis, University of Colorado, 2006

[6] “Noise Figure vs. PM Noise Measurements: A Study at Microwave Frequencies” by Hati, Howe, Walls and Walker, NIST, Proc IEEE Intl Frequency Control Symposium, 2003

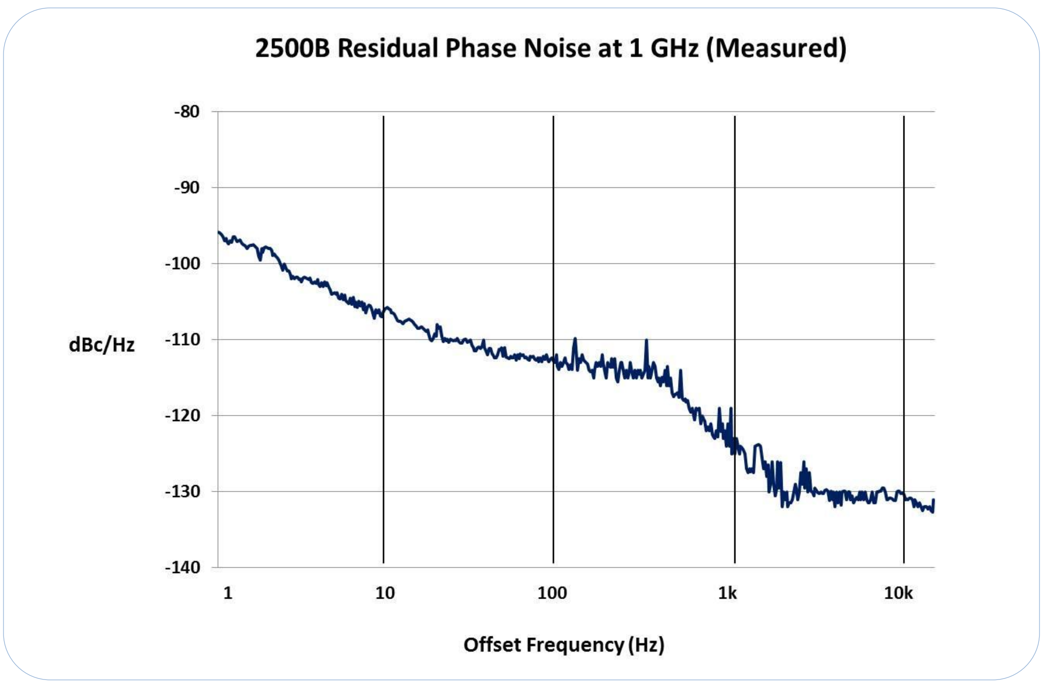
Another way to state the result is:

$$\mathcal{L}_{out}(f) = \mathcal{L}_{in}(f) + \mathbf{SNR} + \mathcal{L}_{residual}(f) \qquad (\text{dBc/Hz})$$

Where SNR is the signal-to-noise ratio and $\mathcal{L}_{residual}(f)$ is the amplifier's residual phase noise. (SNR is flat with frequency).

Absolute vs Residual Phase Noise in Signal Generators

Signal generator phase noise specifications are for absolute phase noise. Absolute phase noise is the phase noise of the RF output signal from the signal generator. Residual phase noise is the amount of phase noise that is attributable to the signal generator after the phase noise of the frequency reference oscillator is subtracted. This makes it possible to predict the degree to which the reference oscillator impacts the total system phase noise when used in conjunction with an external frequency reference, each of which contributes its own phase noise to the total. The phase noise of the external frequency reference usually dominates the close-in phase noise. Mathematically, when there are multiple contributors to phase noise, the total is the square root of the sum of squares (RSS) of the individual contributors. Below is a graph of the 2500B residual phase noise. Note that because the phase noise of the frequency reference has been removed, the close-in phase noise is much lower than it is on the absolute phase noise plots.



Introduction to Leeson's Equation

Suppose that you are designing an oscillator for low phase noise. How would you model the phase noise?

The usual practice today is to use an equation developed by Dr. David B. Leeson just prior to founding California Microwave in the 1968 and reproduced with minor variations in the literature.[7, 8] It was also widely presented by Dieter Scherer and others in the Hewlett-Packard Microwave Symposiums in the late 1970's and 1980's.[9]

Leeson's equation is particularly useful for computer simulation to compare theoretical results to measured data, because it linear and a closed form equation. It also provides an intuitive understanding of the mechanisms contributing to the phase noise.

Leeson's equation:

$$\mathcal{L}(f_m) = 10 \text{ Log} \left(\left[\frac{FkT}{2P_{in}} \right] \left[1 + \frac{f_0^{\ 2}}{(2f_mQ_L)^2} \right] \left[1 + \frac{f_c}{f_m} \right] \right) \quad \text{dBc/Hz}$$

Where:
F = active device noise factor
P_{in} = RF power applied to the resonator (W)
Q_L= resonator loaded Q-factor
f_m = offset frequency (Hz)
f_0 = oscillation frequency (Hz)
f_c = active device $1/f^3$ (flicker FM, 30 dB/decade) corner frequency (Hz)

The first term ($FkT/2P_{in}$) represents the noise floor.

The second term ($f_0^2/4f_m^2Q_L^2$) refers to the loaded Q factor. Unloaded Q = Q_u = f_0/bandwidth.
$1/Q_L = 1/Q_u + 1/Q_E$ where Q_E is dominated by the coupling and the device gain (g_m).

The third term (f_c/f_m) refers to phase perturbations and the flicker effects.

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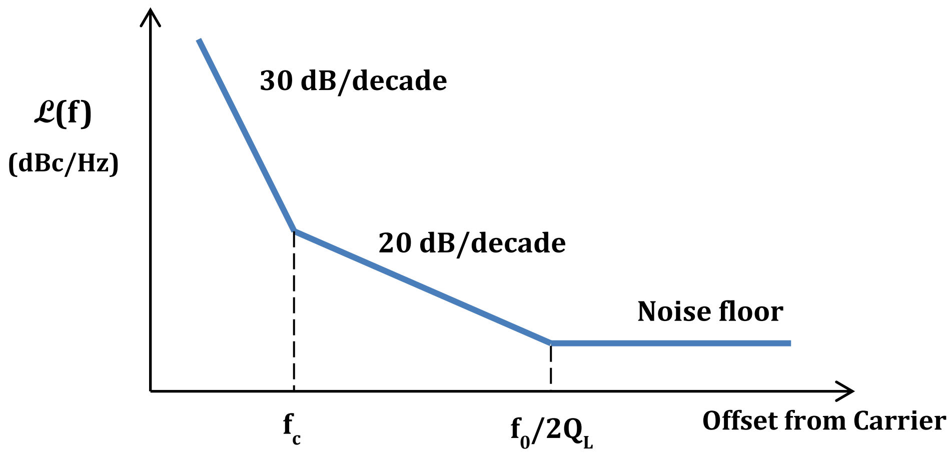
[7] "A Simple Model of Feedback Oscillator Noise Spectrum" by D.B. Leeson, Proc IEEE, 1966
[8] "Frequency Synthesizers: Concept to Product" by Alexander Chenakin, Artech House, 2011
[9] "Design Principles and Test Methods for Low Phase Noise RF and Microwave Sources" by Dieter Scherer, HP RF and Microwave Measurement Symposium, 1978

The result is a model of the oscillator phase noise with three linear regions. The 1/f noise is not included, assumed to be dominated by the $1/f^2$ noise.



From the model (Leeson's equation), it can be seen that the phase noise of an oscillator can be reduced by the following:

1) Lowering the noise floor by choosing an active device with lower noise factor (noise figure) and driving the signal levels at the resonator as high as possible. Post amplifiers (buffer amplifiers) should be low noise also.

2) Lowering the $f_0/2Q_L$ corner frequency by increasing the Q of the resonator and minimize coupling (loading) of the resonator. The literature offers circuit topographies such as the "Push-Push" oscillator that claim no load on the resonator at the resonant frequency.[10,11]

3) Lowering the active device $1/f^3$ (flicker FM, 30 dB/decade) corner frequency by choice of the active device (low noise figure and low flicker noise) and optimizing how it is biased.

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[10] "Wideband Low Phase Noise Push-Push VCO" by Marco Gris, Applied Microwave & Wireless

[11] "Practical considerations on Low-Phase-Noise oscillator Design" by R. Cignanai, et al, CSCC 2002

Conclusion

The literature on phase noise is vast, (much of it quite esoteric) and too numerous to list. This introduction to phase noise is a compilation with the intent of providing an intuitive understanding of phase noise as it applies to microwave signal generators. Additional papers and application notes on phase noise relating to microwave signal generators are available on the Giga-tronics website, www.gigatronics.com.

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